Thermal and Mechanical Stress In Induction Coilguns

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Abstract - **The aim of this paper is to define a procedure for the design of induction coilguns in order to obtain thermal and mechanical stress that do not exceed the allowed values in the sleeve.**

The magnetic vector potential is determined considering a cylindrical sheet current model both for the barrel and the sleeve and solving the related modified Bessel equation. Then the flux, the current density and the propulsive force for each section are determined. By considering the constraints due to mechanical and thermal stress, the maximum muzzle velocity for a one-section launcher is determined.

Supposing that the muzzle velocities in the first and in the last section are established, and assuming that all sections, from the second to the last, work with the same mean slip and the same relative velocity, the number of sections and their length are determined. Moreover the surface current density in the barrel is calculated.

The design criterion is compared with other criteria, and then used to design an 8km/s muzzle velocity launcher.

Index terms – **induction coilguns stresses.**

I. Introduction

Coil-guns seem to promise launch-velocity greater than rail-guns. Indeed, ablative and drag effect are not present in these launchers; and furthermore, there is no current-carrying contact between the barrel and the projectile.

As is well known, the launcher system consists of several sections (barrels), each of which functions as an inductor in a tubular linear induction motor. The projectile mass, a tubular solid conductor, is accelerated as it crosses each section.

Capacitor driven coilguns and generator driven coilguns have been studied adopting full transient and quasi-steady state models [1-8]. The actual operating condition of a coilgun is a transient one, especially for the capacitor driven coilgun, while, a generator driven-coilgun is expected to show relatively smooth variation of the main quantities. Thus, quasi-steady-state models which presume a slow variation in launch system dynamics, can be more suited for the analysis of generator-driven coilguns. Furthermore, their computational simplicity make them attractive for a first design phase of the device.

In this paper a procedure for the design of a generator-driven coilgun is defined to obtain thermal and mechanical stress that do not exceed the allowed values in the sleeve. The model is based on a quasi steady state operating condition including the skin effect in the barrel.

The losses in the sleeve to Joule heating, like sleeve winding losses in a rotating induction motor, increase when the slip increases. To reduce these losses and, therefore, the temperature rise in the sleeve, it is necessary to operate with a small slip. This can be obtained by increasing the feeding current frequency, and consequently the velocity of the traveling magnetic field, from the first to the last section. The design model is based on the assumption that all sections, from the second to the last, work with the same mean slip and the same relative velocity. Then, supposing that the muzzle velocity in the first section is given, we determine the number of sections and their lengths to obtain a desired launch velocity.

II. Mathematical Model

We assume cylindrical current sheets in the barrel and in the sleeve with radii r_1 and r_2 corresponding to the average radii of the corresponding windings. The hypothesis is reasonable because of the quite continuos distributions of the barrel windings along the air gap and because the skin depth is small in the sleeve at the usual operating conditions.

The barrel azimuthal current density K_s can be defined as a travelling wave:

$$K_s = K_s \cos(\omega t - \beta z) \tag{1}$$

where ω is the angular frequency, $\beta = \pi/\tau$ is the wave number and τ the pole pitch.

Neglecting the displacement current, the azimutal component A_ϑ of the magnetic vector potential can be determined by the modified Bessel equation:

$$\frac{d^2 A_\vartheta}{dr^2} + \frac{1}{r}\frac{dA_\vartheta}{dr} - \left(\beta^2 + \frac{1}{r^2}\right) A_\vartheta = 0 \tag{2}$$

with the boundary conditions:

$$H_z(r_1^+) - H_z(r_1^-) = K_s \quad B_r(r_1^+) = B_r(r_1^-) \quad A(\infty) = 0 \tag{3}$$

Manuscript received May 1, 1998,
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where the subscripts $^{+}$ $^{-}$ denote the outside and inside limit values with respect to the radius, H_z is the axial component of the magnetic field and B_r is the radial component of the magnetic flux density.
The average value of the force density N/m^2 acting on the surface of the sleeve in a section is [7]:

$$F_{zm} = 2F_M \frac{s s_c}{s^2 + s_c^2} \tag{4}$$

where F_M and s_c are respectively the maximum force density and the corresponding slip whose value are:

$$F_M = \frac{\mu_o \beta r_1^2 K_1^2(\beta r_1) I_1(\beta r_2)}{4 r_2 K_1(\beta r_2)} K_s^2 \tag{5a}$$

$$s_c = \frac{1}{\mu_o \sigma d v_s \beta r_2 K_1(\beta r_2) I_1(\beta r_2)} \tag{5b}$$

where μ_o, σ, d, v_s are respectively the vacuum permeability, the conductivity and the thickness of the sleeve and the velocity of the travelling wave; K_1 and I_1 are the modified Bessel functions. Defining f as the frequency, the synchronous speed can be obtained as:

$$v_s = 2\tau f \tag{6}$$

III. Multi-section Coilgun Design

Assuming constant initial and final slip for every section (except for the first one where we have initial slip equal to one), the exit and synchronous speed for the i-th section are:

$$v_{ui} = \left(\frac{1-s_2}{1-s_1}\right)^{i-1} v_{u1} \qquad v_{si} = \left(\frac{1-s_2}{1-s_1}\right)^{i-1} v_{s1} \tag{7}$$

where s_1 and s_2 are the initial and final slip in the section. Defining $\Delta s = s_1 - s_2$ the velocity gain in the section is:

$$\Delta v_i = v_{ui} - v_{u(i-1)} = (s_1 - s_2) v_{si} = \Delta s v_{si} \tag{8}$$

From (8) we have:

$$\Delta s = \frac{\Delta v_i}{v_{si}} = \Delta v_{ri} \tag{9}$$

Then, under the assumed hypotheses for the slip, the relative speed variation is constant in every section. Defining v_{uf} as the exit speed and substituting i = N in equation (7), the number of section N results:

$$N = 1 + \frac{\ln \frac{v_{uf}}{v_{u1}}}{\ln \frac{1-s_2}{1-s_1}} \tag{10}$$

The length of the feeding current of every section can be obtained as a function of the length of the first one as:

$$l_i = l_1 k_i = 2\tau k_i \qquad f_i = f_1 \left(\frac{1-s_2}{1-s_1}\right)^{i-1} \tag{11}$$

where k_i is an integer number. Indeed, assuming nearly uniform acceleration, and define the mean slip s_m as:

$$s_m = \frac{s_1 + s_2}{2}$$

the crossing time of every section is:

$$\Delta t_1 = \frac{4\tau}{v_{u1}} \qquad \Delta t_i = \frac{k_i l_1}{(1-s_m) v_{si}}$$

Substituting v_{si} from (7) we have:

$$\Delta t_i = \frac{k_i l_1}{(1-s_m) v_{s1} \left(\frac{1-s_2}{1-s_1}\right)^{i-1}}$$

If we consider as a design criterion a constant crossing time through every section, then we have:

$$\frac{k_i}{\left(\frac{1-s_2}{1-s_1}\right)^{i-1}} = \frac{1}{\alpha} = \text{constant}$$

since k_i is integer we assume:

$$k_i = \text{nearest integer to} \left(\left(\frac{1-s_2}{1-s_1}\right)^{i-1} \frac{1}{\alpha}\right) \tag{12}$$

The frequency of the feeding current of every section can be obtained as a function of the frequency of the first one from:

$$v_{si} = 2\tau f_i; \quad 2\tau = \frac{v_{s1}}{f_1}, \quad \text{then} \quad f_i = \frac{v_{si}}{v_{s1}} f_1 = f_1 \left(\frac{1-s_2}{1-s_1}\right)^{i-1}$$

If for the first section we assume k_1=1, then the length of the section, equal to the length of the sleeve is, 2τ. The total length is:

$$l_t = l_1 \sum_{1}^{N} k_i \tag{13}$$

The time interval corresponding to the crossing of the sleeve in every section is:

$$\Delta t_1 = \frac{4\tau}{v_{u1}} \qquad \Delta t_i = \frac{k_i l_1}{(1-s_m) v_{si}} \tag{14}$$

Defining m as the launch mass, R_2 as the outside radius and l as the length of the sleeve, the average force density, on the sleeve, in the i-th section is:

$$F_{mi} = \frac{m}{2\pi R_2 l} \frac{\Delta v_i}{\Delta t_i} \tag{15}$$

The same quantity can be obtained averaging equation (4):

$$F_{mi} = \frac{1}{\Delta s}\int_{s_2}^{s_1} 2F_{M_i}\frac{s s_{c_i}}{s^2+s_{c_i}^2}ds = \frac{F_{M_i}s_{c_i}}{\Delta s}\ln\frac{s_1^2+s_{c_i}^2}{s_2^2+s_{c_i}^2} \qquad (16)$$

Equating (15) and (16) we can determine the maximum force density in every section:

$$F_{M_i} = \frac{m\Delta v_i}{2\pi R_2 l\Delta t_i}\frac{s_1-s_2}{s_{ci}\ln\frac{s_1^2+s_{ci}^2}{s_2^2+s_{ci}^2}} \qquad (17)$$

The corresponding value of the linear current density is obtained from (5):

$$K_{si} = \sqrt{\frac{4r_2K_1(\beta r_2)F_{M_i}}{\mu_o\beta r_1^2K_1^2(\beta r_1)I_1(\beta r_1)}} \qquad (18)$$

IV. Thermal Stress

Assuming an adiabatic process, the Joule energy in the sleeve during the crossing of the i-th section is:

$$\Delta E_i = \frac{1}{2}\frac{s_m}{1-s_m}m\left(v_{ui}^2 - v_{u(i-1)}^2\right) = m\Delta v_i v_{si}s_m \qquad (19)$$

and the corresponding temperature variation is:

$$\Delta\vartheta_i = \frac{\beta}{c}\Delta v_i v_{si}s_m \qquad (20)$$

where c is the specific heat, β is the ratio between the whole launch mass and the conductive part of the launch mass:

$$\beta = \frac{m}{m_c} \qquad (21)$$

The temperature variation during the crossing of the N sections is:

$$\Delta\vartheta = \frac{1}{2}\frac{\beta}{c}v_{s1}^2\left(1-s_2^2\right) + \sum_2^N\frac{\beta}{c}\Delta v_i v_{si}s_m \qquad (22)$$

where the first term is due to the losses in the first section. Equations (20) and (22) show that the temperature increase is not dependent on the projectile mass. The temperature variation depends on the average slip, on the speed variation, on the synchronous speed. The ratio between the fusion temperature and the actual temperature is:

$$\eta = \frac{\vartheta_f}{\Delta\vartheta} \qquad (23)$$

Defining ϑ_f as the fusion temperature in the conductive part of the sleeve and η_1 a security coefficient the maximum speed that can be obtained with only one section is:

$$v_{u1T} = \sqrt{\frac{2c\vartheta_F}{\beta\eta_1}\frac{1-s_2}{1+s_2}} \cong \sqrt{\frac{2c\vartheta_F}{\beta\eta_1}} \qquad (24)$$

V. Mechanical Stress

The ratio between the maximum value of the force density σ_M and the maximum value of the force density during the crossing of every section is indicated as υ_i:

$$\upsilon_i = \frac{\sigma_M}{F_{Mi}} \qquad (25)$$

We define υ as the value in the last section. The maximum exit speed from the first section is:

$$v_{u1M} = \sqrt{\frac{2\pi R_2 l_1\sigma_M}{\upsilon_1\beta\gamma d r_2}} \qquad (26)$$

Defining γ as the mass density and R_1 the inside radius of the sleeve, the sleeve thickness can be determined as:

$$d = \sqrt{R_1^2 + \frac{m_c}{\gamma\pi l}} - R_1 \qquad (27)$$

To have a uniform current distribution the thickness d should be similar to skin depth δ:

$$\delta = \sqrt{\frac{2\tau}{\sigma s_m\pi\mu v_s}} \qquad (28)$$

where σ and μ are the conductivity and permeability of the sleeve. Defining t the height of the air gap, r_1 and r_2 can be obtained as:

$$r_2 = R_1 + \frac{d}{2} \qquad r_1 = r_2 + t \qquad R_2 = R_1 + d \qquad (29)$$

VI. Results

The described procedure is used for the preliminary design of a multi-section coilgun with an aluminium sleeve with these characteristics:

- initial speed	0
- exit speed	v_{uf} = 8 km/s
- outer radius of the sleeve	R_1 = 2 cm
- specific heat	c = 908 J/kg °C
- fusion temperature	ϑ_f = 658 °C
- mass density	γ=2700 kg/m^3
- conductivity	σ= 1.63x10^{7}S/m
- stress limit	σ_m = 6.9x10^7 Pa

Furthermore we assume: $\eta_1 = \upsilon_1 = 4$, $\beta = 1.3$, and t = 8mm. Substituting N from (10) we have:

$$A = \frac{1-s_2}{1-s_1} = e^{\frac{1}{N-1}\ln\frac{v_{uN}}{v_{u1}}} \qquad (30)$$

and then:

$$s_2 = 1-(1-s_1)A \tag{31}$$

From (31) we have:

$$s_1 > 1-\frac{1}{A} \tag{32}$$

Fig. 1 shows the maximum exit speed from the first sections a function of the launch mass and for several values of the sleeve length.

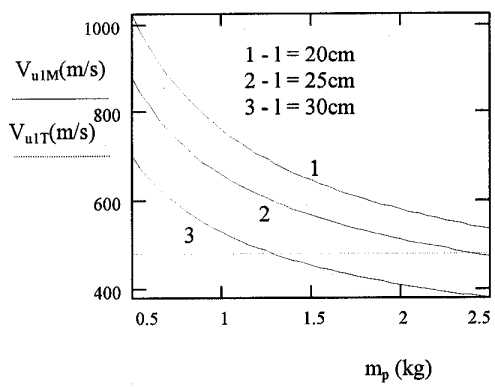

Fig. 1 Maximum exit speed from the first section V_{u1M}, and from the final section V_{u1T} vs. launch mass

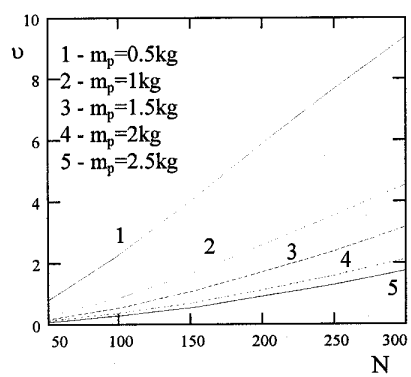

Fig.2 υ as a function of the number of section for several values of the launch mass

The figure shows that for small mass the velocity is mainly limited by thermal stresses (dotted line), while for larger mass the main limit is due to mechanical stress. As the sleeve length increases the limits due to thermal stress becomes more important.

Figures 2 and 3 show the trend of υ and η as a function of the number of sections for different values of the launch mass. The sleeve length equal to the length of the first section is 20 cm. Thermal and mechanical stresses diminish as the number of sections increases. Thermal stress is not dependent on mass, while mechanical stress grows as mass increases. Fig.4 shows the behaviour of η, υ and δ/d as a function of sleeve length, and for m_p=0.5kg and N=200.

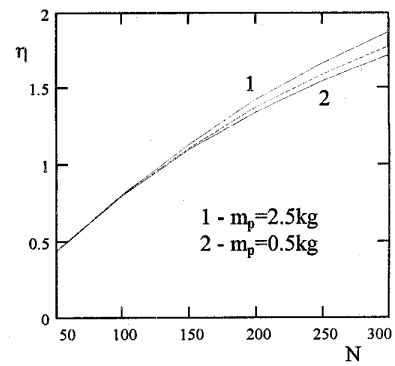

Fig.3 η as a function of the number of section for several values of the launch mass

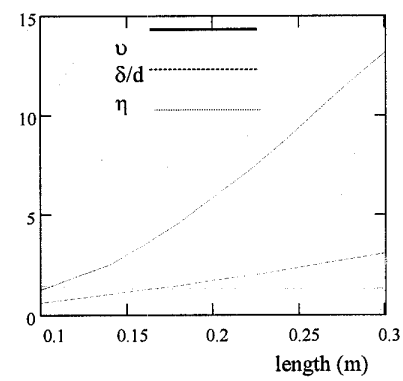

Fig. 4 η, υ, δ and d as as a function of the sleeve length

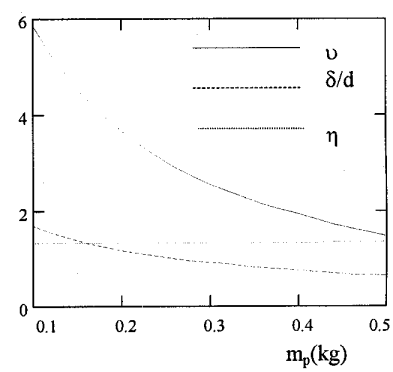

Fig. 5 η, υ, δ and d as a function of the sleeve mass

Mechanical stress diminishes, and skin depth rises, as the sleeve length increases. It is thus advantageous use a longer sleeve with equal launch mass. The thermal stress is not dependent on this quantity. Fig. 5 shows the trends of the same quantities as a function of the launch mass for N=200 sections and a sleeve length of 20cm. The figure confirms that thermal stresses are not dependent on the mass. Furthermore, the figure shows that mechanical stresses rise, and δ/d diminishes with increasing mass.

We consider now a launch mass 0.5Kg and a sleeve 20cm long, with N=200 sections, having an exit speed from the first section vu_1=450m/s.

In Fig. 6 the calculated launcher length and υ value are represented as a function of the parameter α of equation (12). The figure shows that a shorter launcher with higher mechanical stresses is obtained for large α values, while for lower α values a longer launcher and lower stresses result.

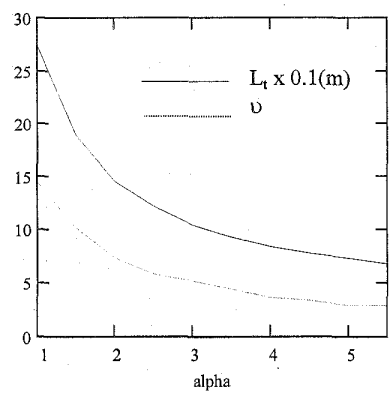

Fig. 6 Length of the launcher and υ values as a function of α

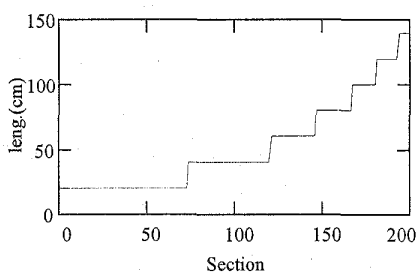

Fig. 7 Length of the sections

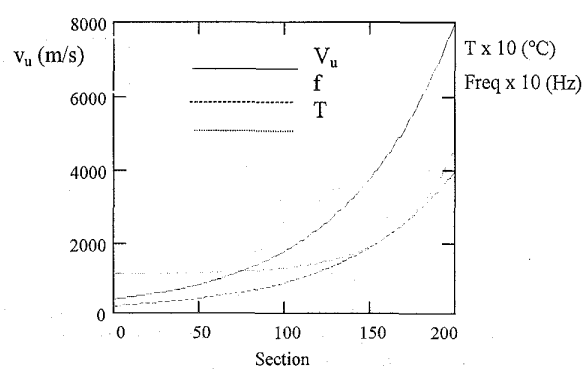

Fig. 8 Frequency, velocity and temperature in the sections

The variation of α allow us to randomize the truncation of the coefficient k_i. In other words we have a random sequence of sections where k_i is rounded to nearest integer towards minus infinity and sections where k_i is rounded to nearest integer towards infinity. Fig. 7 shows the length of the sections corresponding to α=3, υ=5 and l_t=104m. Fig. 8 shows a typical trend of frequency, velocity and temperature in the sections. The behaviour of the force and current densities in the sections are displayed in Fig. 9.

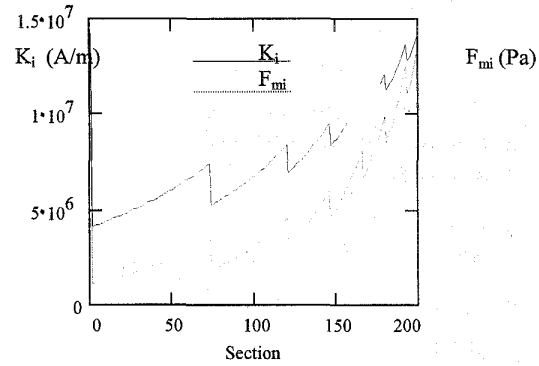

Fig. 9 Linear current density and maximum stress in the sleeve in the sections of the coilgun

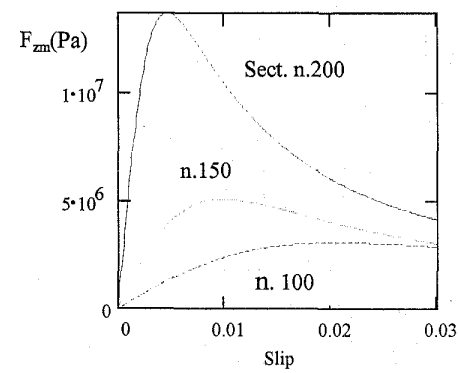

Fig.10 Accelerating force as a function of the slip in some section

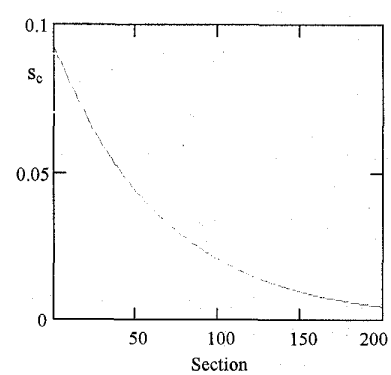

Fig.11 Slip value corresponding to the maximum torque in every section

These trends are common for every simulation. Fig. 9 shows a maximum in the stress and in the current density in the first section. Fig. 10 depicts the behaviour of the force density as a function of the slip in the section 100, 150 and 200; Fig.11 reports the slip corresponding to the maximum force in every section. The figures show that the force rises from the first sections to the last ones, while the slip at the

maximum force diminishes. In other words, the maximum force takes place nearer to the end of the coilgun.

CONCLUSIONS

Under the assumed hypotheses of quasi-stationary conditions, and constant input and output slip in the sections, a design procedure for a multi-section coilgun has been developed. Calculated results have shown that the thermal stress mainly limits the exit speed from the first section for small launch masses while, for larger masses this velocity depends on the geometrical dimensions of the launcher and on the launch mass value. Furthermore it has been shown that:

- The thermal stress in every section is not dependent on the launch mass. This mainly depends on the adopted material, and on the synchronous speed. Thermal stress can be reduced by increasing the number of sections under the same conditions;
- The mechanical stress, for equal exit speed, diminishes as the number of sections increase; furthermore larger sleeves have better performance than longer ones for the same mass.
- The mechanical stress increases with the launch mass.
- The trends of the main quantities of the launcher shows that the thermal and mechanical stress reach the maximum value in the last section.

We note that some of our assumptions are arbitrary (e.g. constant transit time and slip value per section). Actual design choices could lead to accelerators of different appearance, and to modifications of these relationships.

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