Pulsed Coilgun Limits

Stephen Williamson and Alexander (Sandy) Smith
Department of Engineering, University of Cambridge, Trumpington Street, Cambridge, CB2 1PZ

***Abstract*–The design of coilguns is commonly based on a 'try and see' basis using an analytical model to ascertain its performance. This paper examines the use of formal optimisation techniques to explore the performance limitations of pulsed coilgun technology. The optimisation procedure is constrained by the material, mechanical and thermal properties and seeks to determine the coil geometry that achieves the maximum velocity for a given payload. The paper illustrates the maximum velocity achievable for several different payloads as a function of the coilgun length and bore diameter.**

I. INTRODUCTION

Though conceptually simple, coilguns [1, 2, 3, 4] are extremely difficult to design. Present design is largely based on an analytical 'cut-and-try' approach, in which the performance of a trial design is predicted and design parameters adjusted until performance goals are met. Design procedures which make use of formal optimisation techniques have been proposed [5], but despite recent developments they are still enormously expensive to run and give little or no insight into the interdependence of design parameters and performance. Furthermore such procedures can give no useful insight toward performance limits on coilguns. The purpose of this paper is to address this issue.

II. THEORETICAL DEVELOPMENT

A. Kinetic energy

The work contained in this study is based on a simple system of two coaxial air-cored coils carrying currents i_1 and i_2 respectively, shown in Fig. 1. The force developed between these coils is [6]

$$F = i_1 i_2 \frac{dM}{dz} \tag{1}$$

where M is their mutual inductance. For filamentary coils

$$M = M(r_1, r_2, z) = \mu_0 \sqrt{r_1 r_2} \left\{ \left(\frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right\} \tag{2}$$

Manuscript received April 8, 1996.
S Williamson, phone +44-1223 332664, fax +44-1223 332082, e-mail sw@eng.cam.ac.uk;
A C Smith, phone +44-1223 332648 e-mail acs@eng.cam.ac.uk

where
$$k^2 = \frac{4 r_1 r_2}{z^2 + (r_1 + r_2)^2} \tag{3}$$

K(k) and E(k) are complete elliptic integrals of the first and second kind. If the outer coil is stationary and the inner coil is free to move, the work done when it moves from $z = z_1$ to $z = z_2$, is

$$W = \int_{z_1}^{z_2} i_1 i_2 \frac{dM}{dz} dz \tag{4}$$

Assume, now, that i_1 and i_2 are held constant at their maximum values (which may depend on thermal or mechanical limits), then equation 4 becomes

$$W = i_1 i_2 \left[M(z_2) - M(z_1) \right] \tag{5}$$

Let us assume that all of this work is converted to kinetic energy (i.e. ignore mechanical losses, and joule losses in the armature coil) then maximum kinetic energy is achieved when the bracketed term in equation 5 is maximised. The maximum and minimum values of M(z) occur at z = 0 and z = ∞ respectively. $M(\infty) = 0$, therefore

$$KE_{max} = i_1 i_2 M(0) \tag{6}$$

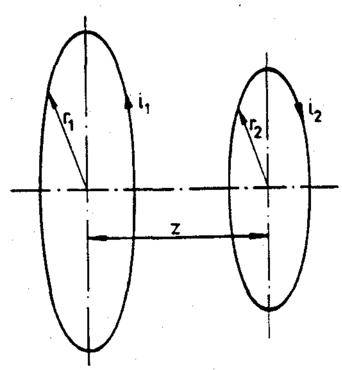

Fig 1 - Filamentary coaxial coils

Equation 6 shows that maximum kinetic energy will be obtained if the coils are initially co-planar and the currents in them are held at their maximum permissible value throughout. In a real (i.e. physical) system the currents cannot be instantaneously switched to, and held at, their maximum values. The current in the armature (i.e. the inner coil), in particular, will vary with time. The purpose of this paper, however, is to determine absolute performance ceilings, and under that condition the assumption of constant

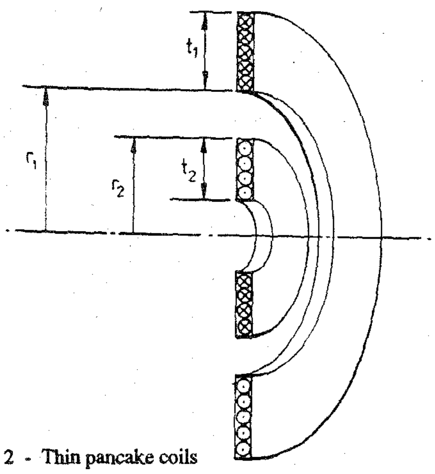

Fig 2 - Thin pancake coils

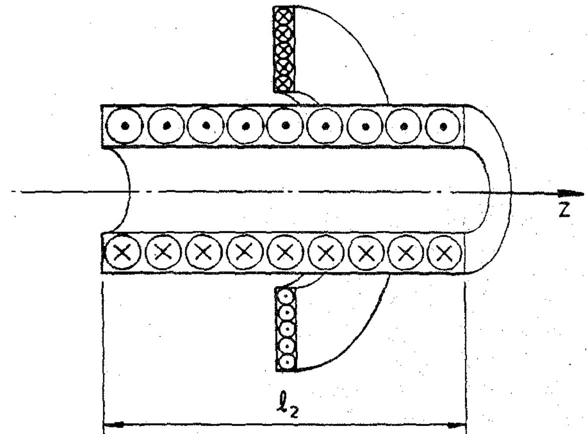

Fig 3 - Armature coil with finite length

maximum current is justified. In Fig. 2 the filamentary coils of Fig. 1 have been replaced by thin pancake coils. The general expression given in equation 6 must still be valid, although the mutual inductance must now be obtained by taking an average over the radial width of the coils. If the coils have N_1 and N_2 turns, respectively, and negligible axial length

$$KE_{max} = i_1 i_2 N_1 N_2 \frac{1}{t_1 t_2} \int_{r_1}^{r_1+t_1} \int_{r_2-t_2}^{r_2} M(0)\, dr_1' dr_2' \tag{7}$$

As a next step assume that the armature coil now has finite length ℓ_2, as shown in Fig. 3. Under the constant maximum current assumption it is a straightforward matter to show that maximum kinetic energy is obtained when the initial position of the armature coil is symmetrical with respect to the pancake coil, as shown in that figure. The integration must now also take place over the length of the armature

$$KE_{max} = i_1 i_2 N_1 N_2 \frac{1}{t_1 t_2 \ell_2} \int_{r_1}^{r_1+t_1} \int_{r_2-t_2}^{r_2} \int_{-\frac{\ell_2}{2}}^{\frac{\ell_2}{2}} M(r_1', r_2', z)\, dr_1' dr_2'\, dz \tag{8}$$

$$= \frac{i_1 i_2 N_1 N_2}{t_1 t_2 \ell_2} S \tag{9}$$

$$\text{where } S = \int_{r_1}^{r_1+t_1} \int_{r_2-t_2}^{r_2} \int_{-\frac{\ell_2}{2}}^{\frac{\ell_2}{2}} M(r_1', r_2', z)\, dr_1' dr_2'\, dz \tag{10}$$

The final stage in the development of the model is to permit the stator coils to have finite length. To do this it is necessary to consider Fig. 4, in which all the stator coils to the left of the centre-line of the armature are energised, whilst all those to the right are not. As the armature moves (to the right in Fig. 4) current is assumed to be instantaneously switched into the armature coils that lie on this centre-line. This may be regarded as equivalent to sequentially exciting a row of incremental 'pancake' coils in the stator, as the armature centre passes them. Each additional incremental excitation produces a contribution to the maximum kinetic energy given in equation 9. That equation is therefore valid for the complete launcher, provided N_1 is now interpreted as the total number of turns in the stator.

Equation 9 is still inconvenient for use, because it contains currents and turns numbers, which may often be scaled to suit supply conditions. To circumvent this, we note that

$$i_1 N_1 = J_{c1} k_1 t_1 \ell_1 \tag{11}$$

$$i_2 N_2 = J_{c2} k_2 t_2 \ell_2 \tag{12}$$

where J_{c1} and J_{c2} are the current densities in the stator and armature conductors respectively. k_1 and k_2 are corresponding fill factors (ie ratio of copper section to winding section in a coil). ℓ_1 and ℓ_2 are the axial lengths of the stator and armature.

Substituting 12 and 11 into 9,

$$KE_{max} = J_{c1} J_{c2} k_1 k_2 \ell_1 S \tag{13}$$

If it is assumed that an inert projectile of mass m is accelerated *from rest* by an armature of mass αm to velocity v, then the kinetic energy gained is

$$KE = \tfrac{1}{2}(1+\alpha)\, mv^2 \tag{14}$$

where α is the ratio of armature mass to the projectile mass.

Equating equations 13 and 14, to obtain the maximum velocity

$$(1+\alpha)\, mv_{max}^2 = 2 J_{c1} J_{c2} k_1 k_2 \ell_1 S \tag{15}$$

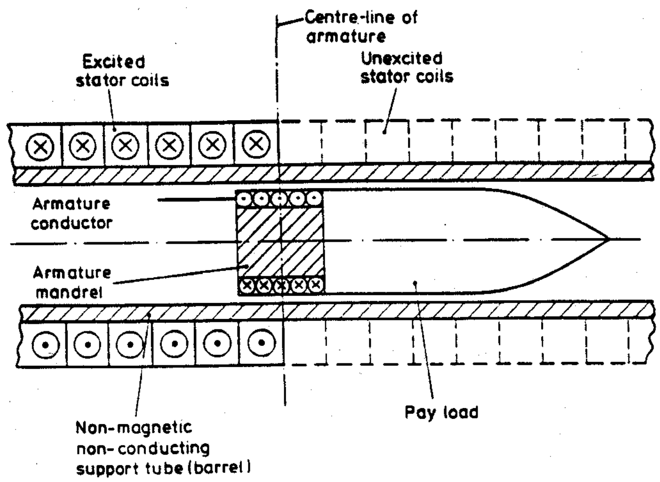

Fig 4 - Stator coil and armature excitation

B. Thermal limits

Although the above analysis assumes that once excited, the currents in the stator windings persist, the mutual inductance between two coils falls rapidly as the distance between them increases. We may safely assume that even in this ideal coil gun, the current in a given stator coil will be reduced to zero when the armature has passed it by a distance equivalent to two or three diameters. Furthermore, being stationary the stator coils are more readily cooled, if necessary. It will therefore be assumed that thermal limits will not be approached on the stator. The armature currents, on the other hand, are required to persist for the entire time that the armature is inside the bore, and for a short distance (again two or three diameters) beyond the end of the bore. Assuming that the armature heats adiabatically, the rate of temperature rise in the armature copper is

$$\frac{d\theta}{dt} = \frac{J_{c2}^2 \rho}{dC_p} \tag{16}$$

C_p is the specific heat of copper, ρ its resistivity, and d its density. As the current in the armature has been assumed to be constant, the temperature rise obtained in time T is given by

$$\theta = \frac{J_{c2}^2 \rho T}{dC_p} \tag{17}$$

where it is assumed that the parameters are not temperature dependent.

C. Mechanical Stresses

1) Stator Coils: The axial flux density at the inside surface of the stator is obtained by the long solenoid approximation as

$$B_{z1} = \mu_o J_{c1} k_1 t_1 \tag{18}$$

The radial stress that this produces is

$$P_s = \frac{1}{2\mu_o} B_{z1}^2 = \frac{\mu_o}{2}\left[J_{c1} k_1 t_1\right]^2 \tag{19}$$

Using the standard approximation for a long solenoid that the magnetic field outside the solenoid is negligibly small, the radial stress there is also approximately zero. The hoop stress in the stator varies in magnitude across the radial thickness of the stator, with the maximum occurring at the inner surface. The magnitude of this maximum is

$$\sigma_{max} = \frac{P_s\left(r_1^2 + (r_1 + t_1)^2\right)}{t_1(2r_1 + t_1)} \tag{20}$$

This maximum stress must not exceed the yield strength of copper (σ_y)

$$\sigma_y \geq \frac{P_s\left(r_1^2 + (r_1 + t_1)^2\right)}{t_1(2r_1 + t_1)} \tag{21}$$

Substituting from equation 19 into equation 21

$$\sigma_y \geq \frac{\mu_o t_1\left(r_1^2 + (r_1 + t_1)^2\right)}{2(2r_1 + t_1)} \left(J_{c1} k_1\right)^2 \tag{22}$$

This equation may be re-arranged to determine the maximum current density in the stator coils

$$J_{c1(max)} = \frac{1}{k_1}\sqrt{\frac{2\sigma_y(2r_1 + t_1)}{\mu_o t_1\left(r_1^2 + (r_1 + t_1)^2\right)}} \tag{23}$$

ii. Armature: Assuming that the clearance between the stator coils and the armature coils is small, the flux density at the outer surface of the armature is the same as that at the inside surface of the stator (i.e. B_{z1} in equation 18). The corresponding radial stress (i.e. P_s in equation 19) therefore also applies. The axial flux density at the inner surface of the armature is given approximately as

$$B_{z2} = \mu_o (J_{c1} k_1 t_1 - J_{c2} k_2 t_2) \tag{24}$$

The radial stress acting on the inner surface of the armature is therefore

$$P_a = \frac{1}{2\mu_o} B_{z2}^2 = \frac{\mu_o}{2}\left[J_{c1} k_1 t_1 - J_{c2} k_2 t_2\right]^2 \tag{25}$$

The hoop stress in the armature varies across its radius with maximum stress appearing on either the inner or outer surfaces, depending on the relative magnitudes of P_a and P_s.

For the inner surface, $|\sigma_{in}| \le \sigma_y$,

$$\sigma_{in} = \frac{2P_s r_2^2 - P_a\left((r_2 - t_2)^2 + r_2^2\right)}{\left[r_2 - t_2\right]^2 - r_2^2} \tag{26}$$

For the outer surface, similarly $|\sigma_{out}| \le \sigma_y$, where

$$\sigma_{out} = \frac{P_s\left(r_2^2 + (r_2 - t_2)^2\right) - 2P_a (r_2 - t_2)^2}{\left[r_2 - t_2\right]^2 - r_2^2} \tag{27}$$

The armature also imparts the accelerating force to the projectile. Assuming that this force is imparted through the cross-section of the armature copper, then

$$P_t \pi\left(r_2^2 - (r_2 - t_2)^2\right) = \frac{mv}{T} \tag{28}$$

P_t, the axial stress, must be less than the yield stress of copper, therefore

$$\frac{mv}{\pi\left(r_2^2 - (r_2 - t_2)^2\right)T} \le \sigma_y \tag{29}$$

III. Procedure

The maximum armature current is assumed to be fixed by thermal considerations (via equation 17), whereas the maximum stator current is determined by limitations on the hoop stress in the stator coils (via equation 23). Substituting from these two equations into equation 15

$$(1+\alpha)\, mv_{max}^2 = 2\sqrt{\frac{2\sigma_y (2r_1 + t_1)}{\mu_o t_1\left(r_1^2 + (r_1 + t_1)^2\right)}}\sqrt{\frac{dC_p\theta}{\rho T}}\, k_2 \ell_1 S \tag{30}$$

As it is necessary for the integrity of the armature to be maintained only for the duration of the acceleration, the time T taken for the armature copper to reach its melting temperature (θ) may be equated to the time taken for the armature to pass along the barrel,

i.e. $$\ell_1 = \frac{vT}{2} \tag{31}$$

Substituting equation 31 into equation 30, to eliminate T, and rearranging

$$v_{max} = \left\{\frac{4\sigma_y (2r_1 + t_1)\ell_1 dC_p \theta k_2^2 S^2}{\mu_o t_1\left(r_1^2 + (r_1 + t_1)^2\right)\rho m^2 (1+\alpha)^2}\right\}^{1/3} \tag{32}$$

Equation 32 may be used to calculate the maximum velocity obtainable from a launcher of given dimensions (r_1, r_2, t_1, t_2, and ℓ_1 fixed) for a projectile of given mass (m) and armature of given mass (αm). When this velocity has been determined equations 23 and 17 enable the corresponding stator and rotor current densities, J_{c1} and J_{c2} to be determined. These, in turn, enable the radial stresses produced on the outside of the armature to be calculated via equation 25 and the armature hoop stress limits to be checked via equations 26 and 27. Finally the transit time is obtained through equation 31, and the axial stress checked via inequality 29.

The starting point of the procedure by which performance limits are determined is to assume

(i) Bore (D_b)
(ii) Axial stator length (ℓ_1)
(iii) Projectile mass (m)
(iv) Thickness for the launch tube or barrel (t_b)

These are the principal independent design variables. It is assumed that the radial clearance between the armature and bore is small, and this is set to 0.5 mm in all that follows. This has the effect of fixing r_2, the outside radius of the armature (fig.2)

$$D_b = 2r_2 + 0.001 \tag{33}$$

The stator coils are assumed to fit snugly on to the outside of the barrel, and this determines r_1

$$D_b + 2t_b = 2r_1 \tag{34}$$

The parameters which remain are the stator thickness (t_1), the armature thickness (t_2), and the overall weight of the armature (αm). The axial length of the armature (ℓ_2) is obtained from the equation for the armature mass

$$\alpha m = \pi\left(r_2^2 - (r_2 - t_2)^2\right)k_2\, d\, \ell_2 \tag{35}$$

The goal is to determine the coil thicknesses (t_1 and t_2) that produce the maximum projectile velocity, for a given weight of armature (αm), without exceeding the material stress limits. This may conveniently be achieved using a standard multivariable optimisation procedure, in which the material constraints are imposed through appropriate penalty functions. The optimisation procedure undertakes a multi-dimensional search to determine the maximum projectile velocity without violating any of the material constraints specified in Table 1. In most instances, the maximum velocity corresponds to one or more of these constraints being met. It is, however, difficult in this situation to make a general statement on which of the various material limits is critical to a particular coilgun with a certain armature weight (αm). Relaxation of the material constraints, however, will lead to improvements in the maximum velocity. The triple integral S, given by equation 10, may be evaluated efficiently for each set of design variables generated by the optimisation subroutine by means of Gaussian integration. The elliptic integrals present in equation 2 are evaluated using polynomial expansions [7].

IV. Results

Numerical studies have been made for 50 mm and 150 mm bore coilguns, launching payloads of 1 kg, 2 kg, and 25 kg (150 mm bore only). The assumed physical constants used in the simulations are defined in Table 1. The optimisation procedure determines a new set of coilgun dimensions ($t_1$and t_2) and the axial length indirectly from equation 35 for each and every armature weight (αm) to maximise the projectile velocity. For brevity, these are not included in the results.

Figure 5 shows the maximum velocity achievable with a 50 mm bore, and a projectile of 1 kg . The abscissa gives the weight of the armature (= αm, where m is the mass of the electromagnetically inert projectile). The figure shows that for each length of launcher there is a definite armature mass beyond which the performance deteriorates. The maximum kinetic energy imparted to a 1 kg projectile is 150 kJ, 330 kJ, and 442 kJ by launchers of 2 m, 4 m, and 6 m length, respectively. These maximum energies are approximately proportional to the length of the launcher. Figure 6 shows v_{max} versus α for a 50 mm bore launcher, and a 2 kg projectile. The curves in this figure are essentially similar to those in Fig. 5, but the maximum kinetic energies achievable are now 178 kJ, 392 kJ, and 517 kJ for launchers of 2 m, 4 m, and 6 m. These are significantly different from the corresponding maximum energies achieved with a 1 kg projectile, suggesting that for a given bore and launcher length, there will be an optimum projectile mass that will maximise kinetic energy. The kinetic energies are again approximately proportional to launcher length.

Figures 7 and 8 repeat the above exercise for a launcher of 150 mm bore diameter. The maximum kinetic energies achievable with a 1 kg projectile are 165 kJ, 391 kJ, and 632 kJ, whereas with a 2 kg projectile they are 220 kJ, 563 kJ, and 937 kJ. These values correspond to launcher lengths of 2 m, 4 m, and 6 m respectively. It is clear that at this increased diameter the approximate linear relationship between launcher length and kinetic energy is no longer valid. Figure 9 shows v_{max} versus α for a 25 kJ projectile. The kinetic energies achieved with 2 m, 4 m, and 6 m launchers are 523 kJ, 1.30 MJ, and 2.27 MJ, respectively. Figure 10 gives a comparison between 2 m launchers of differing bore diameter. This figure indicates that with a short launcher and relatively light projectiles there is little to be gained by increasing the bore diameter. Fig. 11 shows that if the length of the launcher is increased to 6 m, then increasing the bore diameter begins to give a more definite improvement in capacity.

All of the proceeding performance graphs relate to launch tubes of zero width. In a practical application it is expected that the coils will be mounted on the outside of an inert fly-way tube of finite thickness. The effect of the presence of such a tube is investigated in Figs. 12 - 15. Fig. 12 indicates that the launch tube will not produce a deterioration of the performance of a 2 m long gun of 50 mm bore, provided a heavier armature is employed. A 6 m long gun of the same diameter, however, will suffer from some deterioration in performance, as shown in Fig. 13. Figs. 14 and 15 show that the presence of the launch tube will produce little deterioration in the performance of a 150 mm bore launcher.

V. Discussion

This paper has made use of an idealised coil gun to explore the limits of pulsed coilgun performance. The principal idealising assumptions that have been made are:

(i) Current switches instantaneously into the stator coils;
(ii) Current in the armature coil is sustained throughout the launch;
(iii) Friction and aerodynamic drag are negligible;
(iv) Material properties remain constant throughout the launch.

It is difficult to assess the general effect of relaxing all of these assumptions - except to state that a reduction in performance will be obtained. It would be possible, however, to investigate specific scenarios using the model developed in this paper as the foundation of the investigation.

The 'fill factors' k_1 and k_2 have been set at 0.5 to obtain the results presented in the paper. Higher fill factors might prove possible in practice. This would have a beneficial effect on performance. Again, such benefits are difficult to assess, because of the stress limits discussed in section 2.3.2.

The launcher configuration considered in this paper is one in which the armature is pushed by the stator excitation. An alternative form of launcher uses a push/pull configuration in which stator currents ahead of the armature pull it while those behind push it. Whilst such a configuration has not been examined by the authors, they believe that the techniques developed in this paper are eminently suitable for such a purpose.

Density of copper	(d)	8933 kg m^{-3}
Stator fill factor	(k1)	0.5
Armature fill factor	(k2)	0.5
Specific heat of copper	(Cp)	430 J kg^{-1} K^{-1}
Maximum temperature rise	(θ)	1000 K
Resistivity of copper	(ρ)	6.0 x 10^{-8} Ωm
Yield strength of copper	(σ_y)	6.0 x 10^7 Nm^{-2}

TABLE I : MATERIAL AND OTHER PROPERTIES

References

[1] J A Andrews "Coilgun structures" *IEEE Transactions on Magnetics* 29(1), pp 637 - 642, January 1992.

[2] J A Andrews & J R Devine "Armature design for coaxial induction launchers" *IEEE Transactions on Magnetics* 27(1), pp 639 - 643, January 1991.

[3] M D Driga & W F Weldon "Induction launcher design considerations" *IEEE Transactions on Magnetics* 25(1) pp 153 - 158, January 1989.

[4] J L He, E Levi, Z Zabar & L Birenbaum "Concerning the design of capacitively driven induction coil guns" *IEEE Transactions on Plasma Science* 17(3) pp 429 - 438, January 1989.

[5] S Williamson & J R Smith "Application of minimisation algorithms in electrical engineering" *IEE Proceedings Part B* 127(8) pp 258 - 530 November 1980.

[6] S Williamson & P J Leonard "Analysis of air-cored tubular induction motors " *IEE Proceedings Part B* 133(4) pp 285 - 289 July 1986.

[7] M Abramowitz & A Stegun "Handbook of Mathematical Functions" *Dover Publications Inc* New York, 1965.

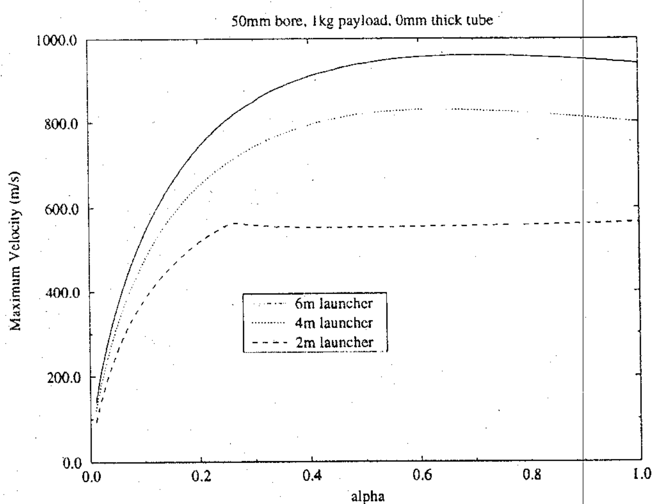

Fig 5 - Maximum Velocity versus alpha

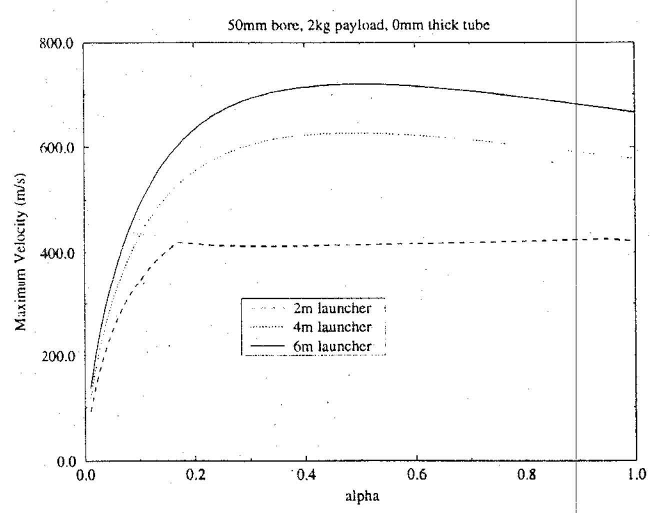

Fig 6 - Maximum Velocity versus alpha

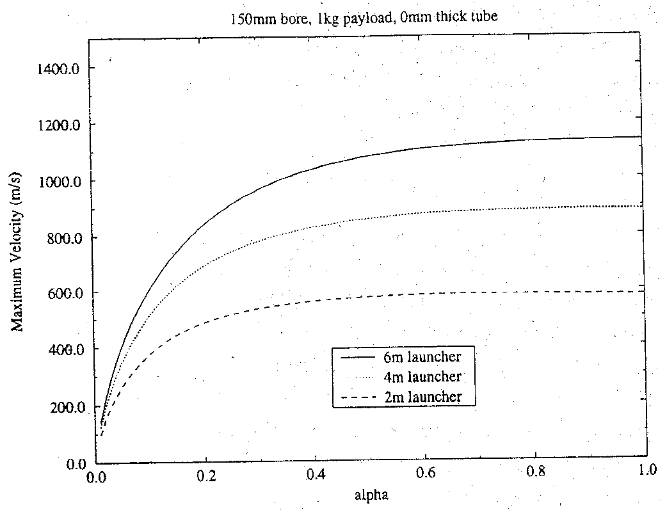

Fig 7 - Maximum Velocity versus alpha

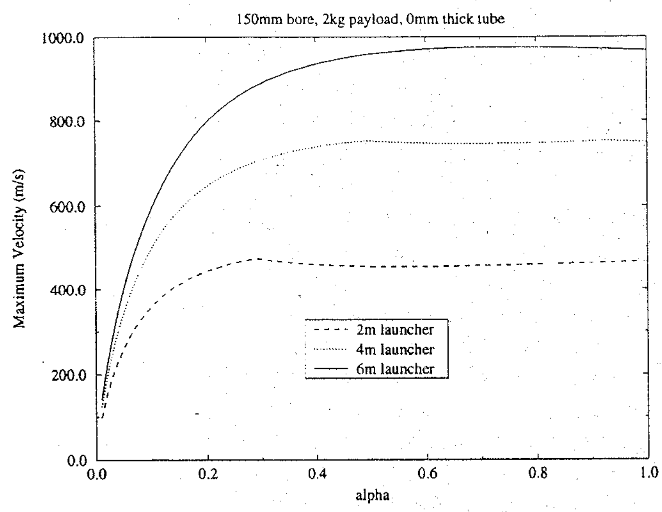

Fig 8 - Maximum Velocity versus alpha

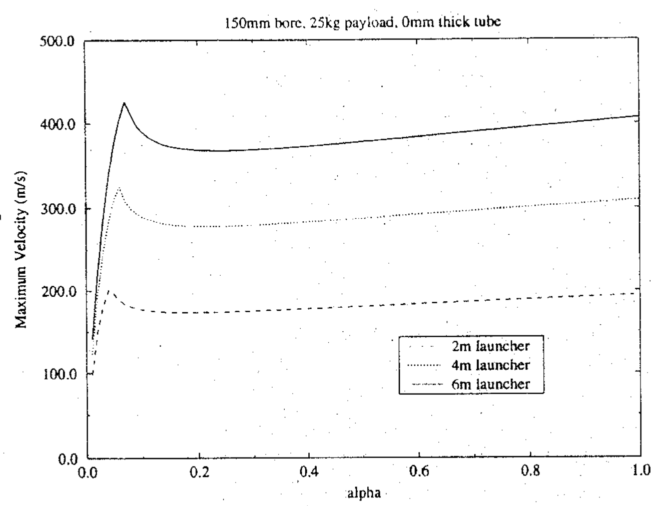

Fig 9 - Maximum Velocity versus alpha

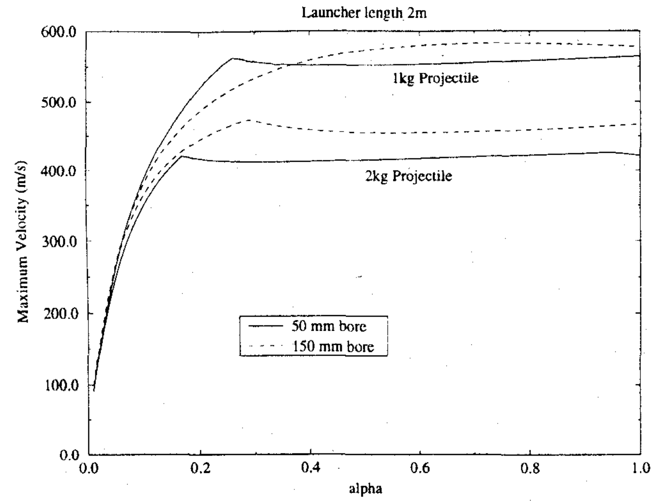

Fig 10 - Maximum Velocity versus alpha

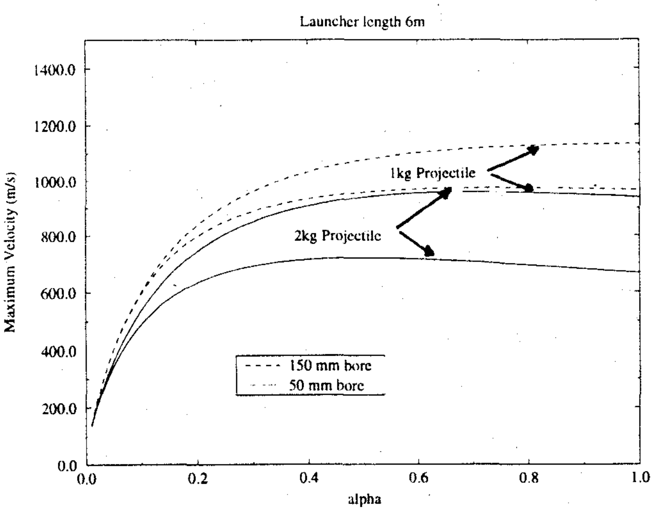

Fig 11 - Maximum Velocity versus alpha

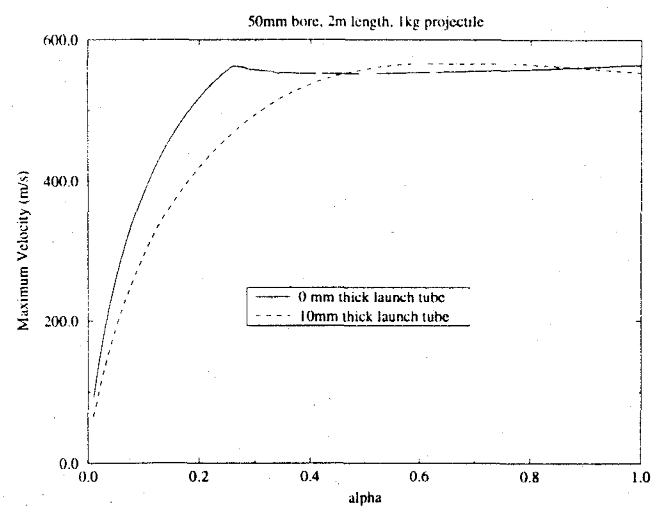

Fig 12 - Maximum Velocity versus alpha

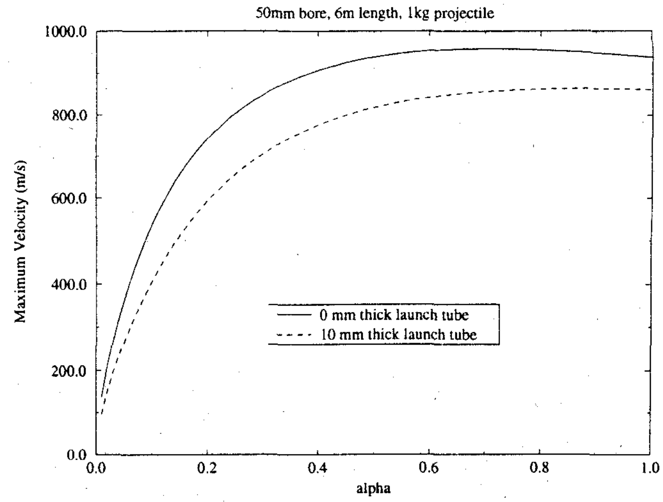

Fig 13 - Maximum Velocity versus alpha

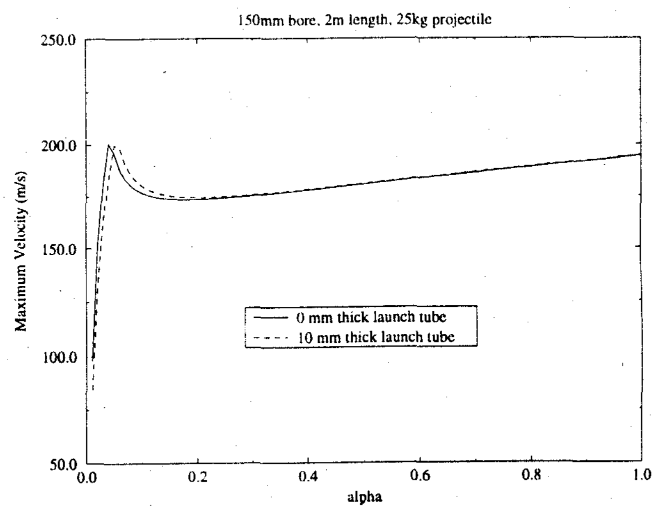

Fig 14 - Maximum Velocity versus alpha

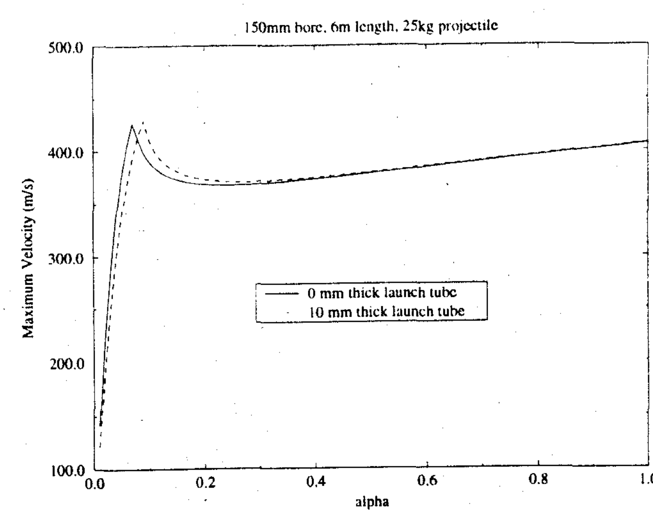

Fig 15 - Maximum Velocity versus alpha